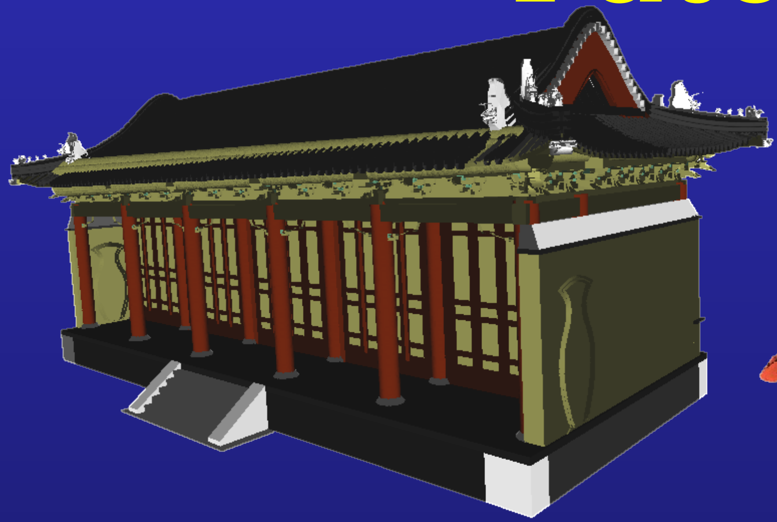
Budget Sampling of Parametric Surface Patches



Jatin Chhugani and Subodh Kumar

Johns Hopkins University

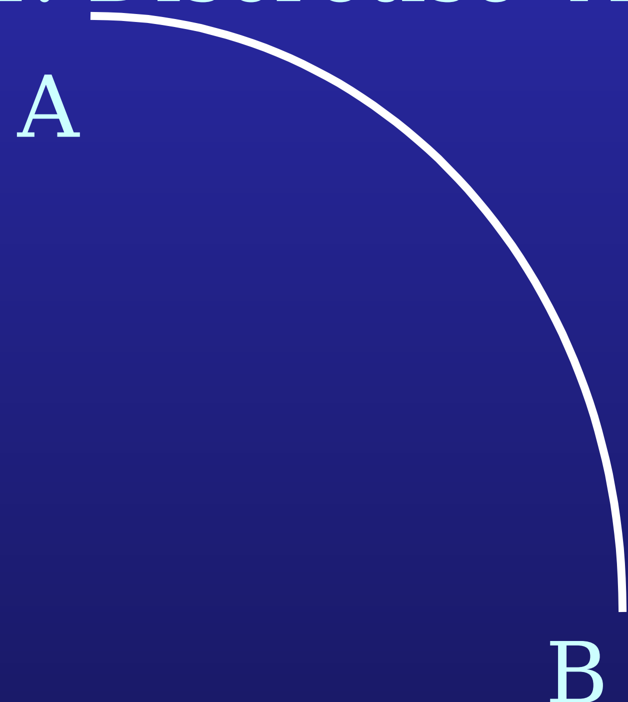
Motivation

- Sampling a continuous surface into discrete points
  - Rendering as triangles or points
  - FEM/BEM analysis for physics based computation
  - Collision detection
- What criteria to satisfy?
  - Inter-sample distance
  - Deviation from the actual surface
- How to choose the best ‘N’ samples ?
- Relationship between the best ‘N’ and the best ‘N+1’ samples?

Example

Problem1: Discretise 'AB' into 5 points?

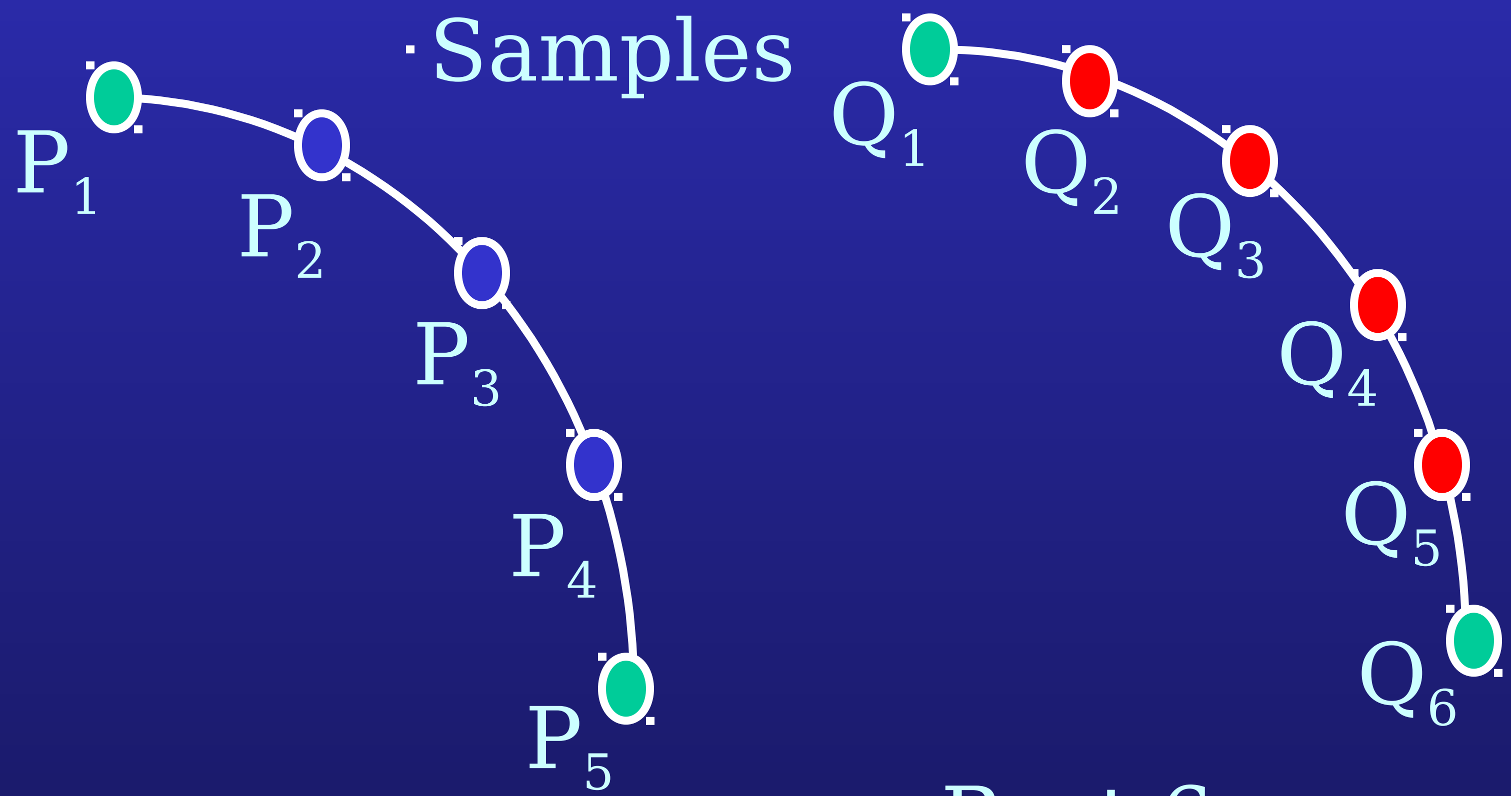
Problem 2: Discretise 'AB' into 6 points?



Example

Common Samples

Different Samples



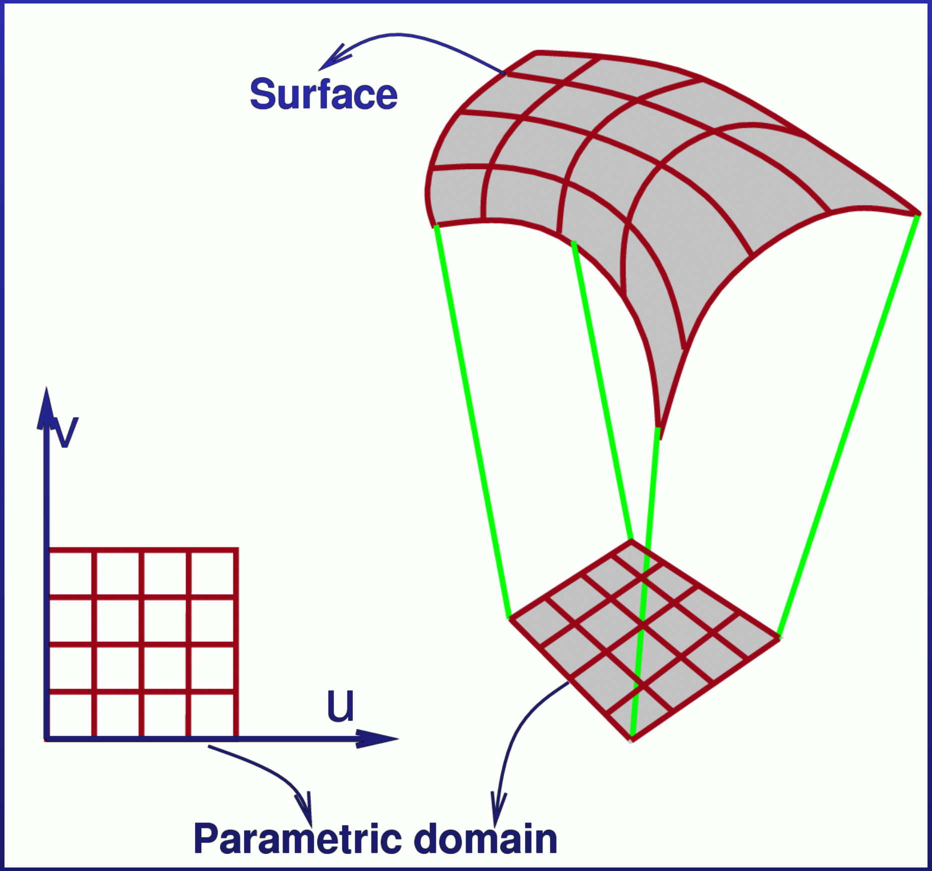
Best 5 samples

Best 6 samples

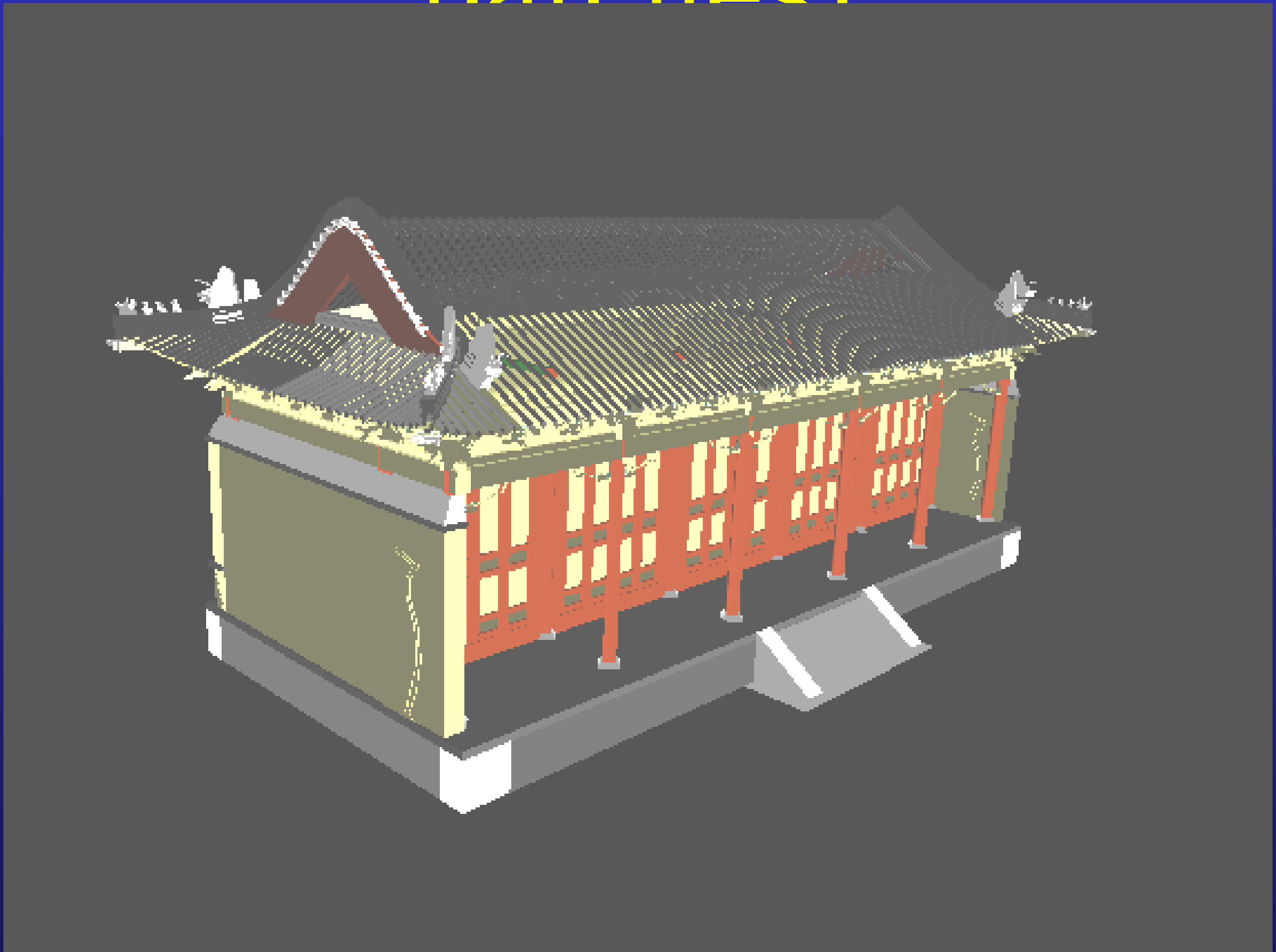
Application of Spline Models

- CAD/CAM , Entertainment Industry
- Medical Visualization
- Examples
  - Submarines
  - Animation Characters
  - Human body especially the heart and brain

Splines



Garden Model (38,646 patches)



Interacting with Spline Surfaces

- Interactive Spline Rendering:
  - Need to update image 20-30 times per second
  - Bound on the number of primitives that can be rendered per second
- Interactive Collision Detection:
  - Need to compute collisions 1000 times per second
  - Bounded CPU/GPU time
  - Upper bound on the number of collision tests per second

Interacting with Spline Surfaces

- Interactive Spline Rendering:
  - Need to update image 20-30 times per second
  - Bound on the number of primitives that can be rendered per second
- Interactive Collision Detection:
  - Need to compute collisions 1000 times per second
  - Bounded CPU/GPU time
  - Upper bound on the number of collision tests per second

Upper Bound on the number of primitives that can be handled per frame

Issues

1. Given a threshold (in terms of number of primitives), how to distribute it amongst various parts of the model?

2. What criteria need to be satisfied?
   - Plausible image (Minimize artifacts)
   - Accurate image (or bounded error, quantification if possible)
   - Bounded Computation Time

Problem Statement

Given a set of surface patches $\{F_i\}$, and total number of primitives (C), allocate C_i to each patch ensuring fairness.

Fairness: Minimize the projected screen-space error of the whole model.

Questions:

1. How to compute C_i to minimize the deviation of samples from surface?
2. For rendering applications, how to render these primitives?

Rendering Splines

- Ray tracing
  - J. Kajiya ['82], T. Nishita ['90], J. Whitted ['79]
- Pixel level surface subdivision
  - E.Catmull ['74], M. Shantz ['88]
- Scan-line based
  - J. Blinn ['78], J. Lane ['80], J. Whitted ['78]
- Polygonal Approximations
  - Abi-Ezzi ['91], Filip ['86], S. Kumar ['96, '97, '01]

Polygonal Approximations

- Produce accurate color and position only at the vertices of the polygons (triangles)
- Computationally intensive to figure out tessellation parameters
- Maintain expensive data-structures with substantial per-frame update costs
- May lead to a large number of small screen-space triangles

Point-Based Rendering

- Introduced by Levoy and Whitted ['85]
- Explored further by Dally ['98], Rusinkiewicz ['00], Pfister ['00], Stamminger ['01]
- Decompose surface into nominally curved \`elements\` which follow the surface more closely, Szeliski ['92], Witkin ['94], Kalaiah ['01]
- Shaded well using algorithms by Zwicker ['01], Kaliah ['02], Adamson ['03]

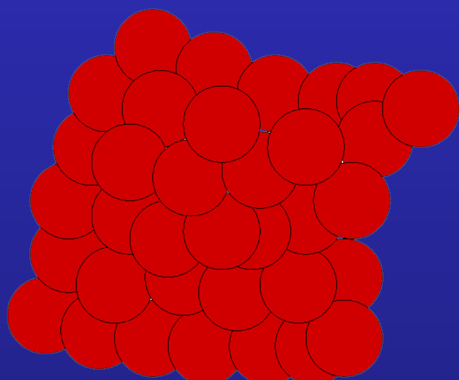
Point-Based Rendering

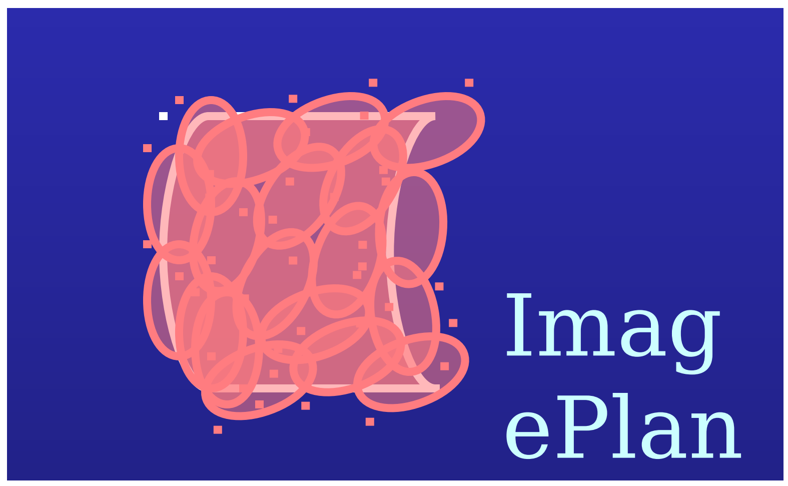
- No need to maintain topological information
- Lower update costs as compared to triangle-based rendering for zoomed-out views
- Less beneficial for zoomed-in views

Attributes of each primitive (for Point-Based Rendering)

- Position (x,y,z).
- Normal (Nx, Ny, Nz).
- Color.
- Size / Shape ?

Spheres as primitives



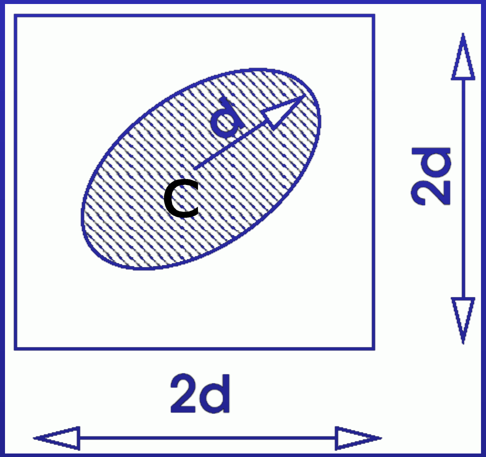


Spheres on the patch in Object Space

Projection of Spheres with no holes

Every point on the surface inside at least one sphere.

Rendering Spheres



1. Compute the maximum deviation (d) of the projected surface from projection of the center (**C**).

2. Draw a square splat of size 2d centered at C.

Our approach

1. Pre-Sampling:

- Progressively compute ordered list of samples on the domain of each patch.
- Each sample associated with a sphere centered on its corresponding point in 3D.
- The radius of the sphere decreases as more points are added.

Our approach

2. View-dependent Point Selection:

- Compute the screen-space error for every patch.
- Compute the scaling factor for every patch.
- Compute the corresponding object-space error.
- Search for this value in the sorted list of error values.
- Render the corresponding samples with a certain point-size.

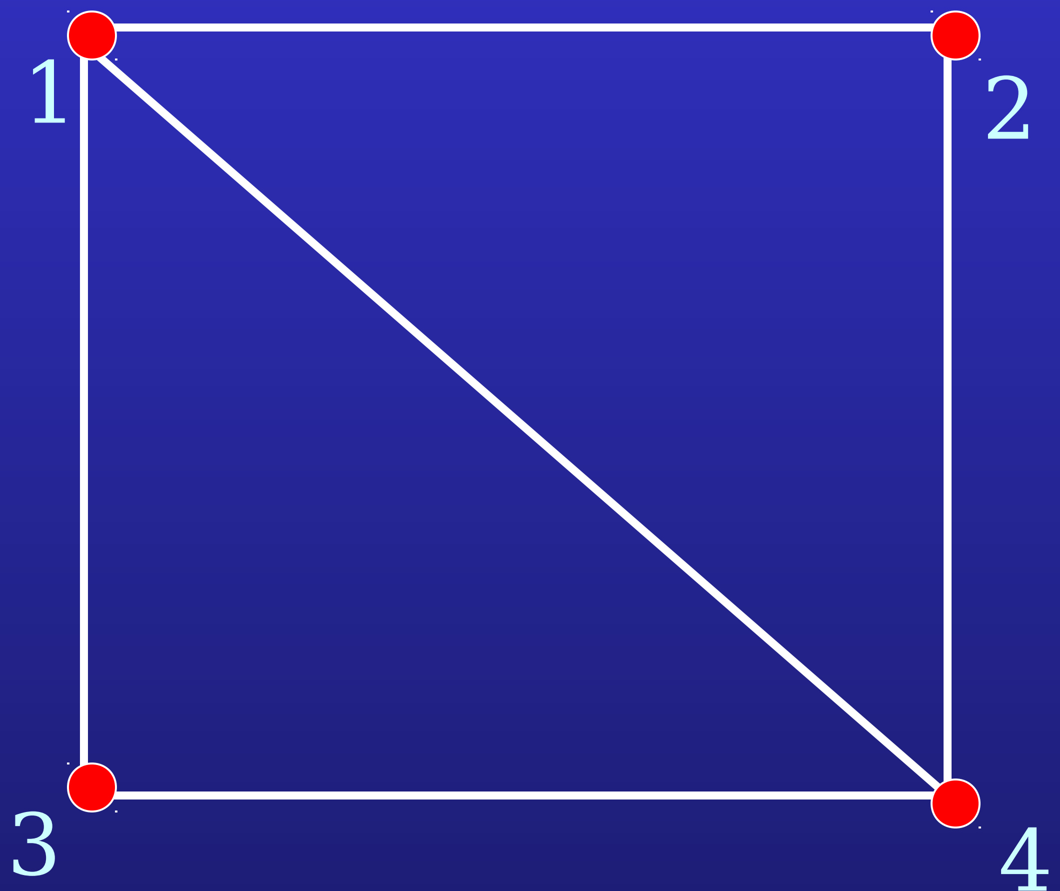
Pre-Computation

Sampling the Domain Sp

Pre-Sampling

- Start with the minimal sample set (e.g. the four corners) in the domain.
- Generate the 2D Delaunay triangulation.

Pre-Sampling

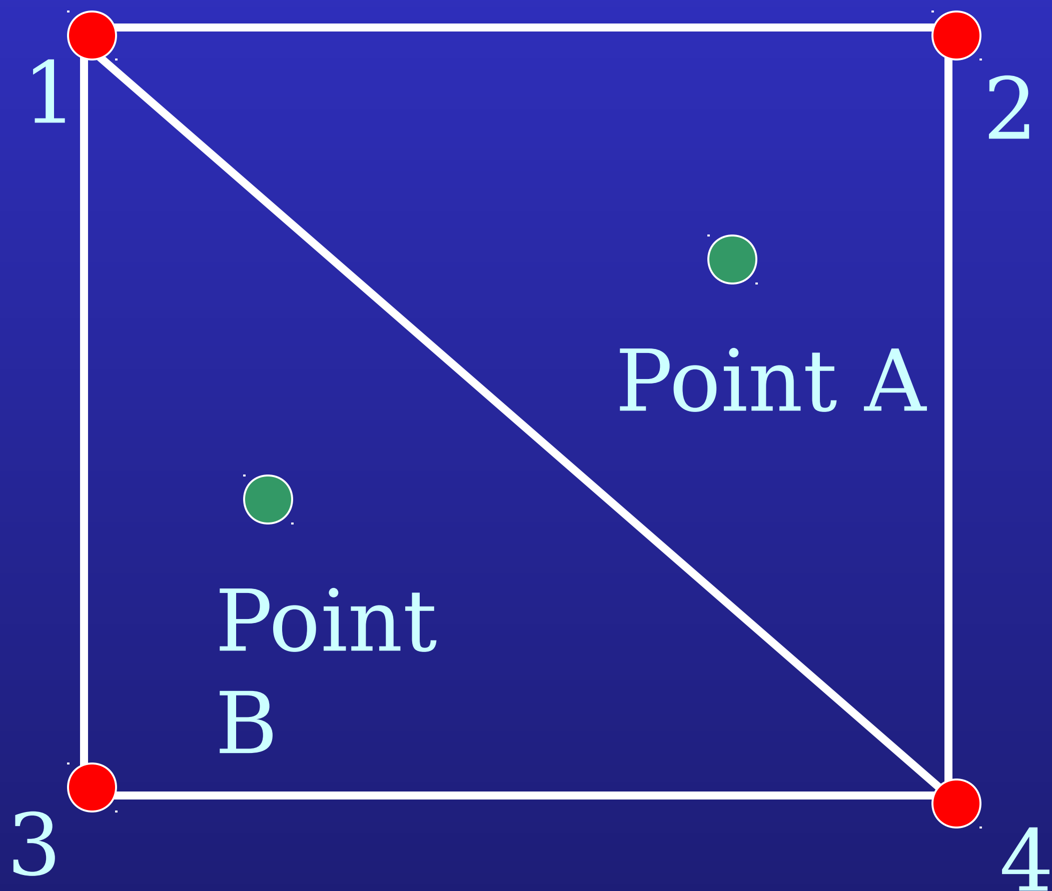


Domain Space

Pre-Sampling

- Start with the minimal sample set in the domain.
- Generate the 2D Delaunay triangulation.
- Compute center and radius of the circumscribing spheres for each triangle (in 3D).

Pre-Sampling

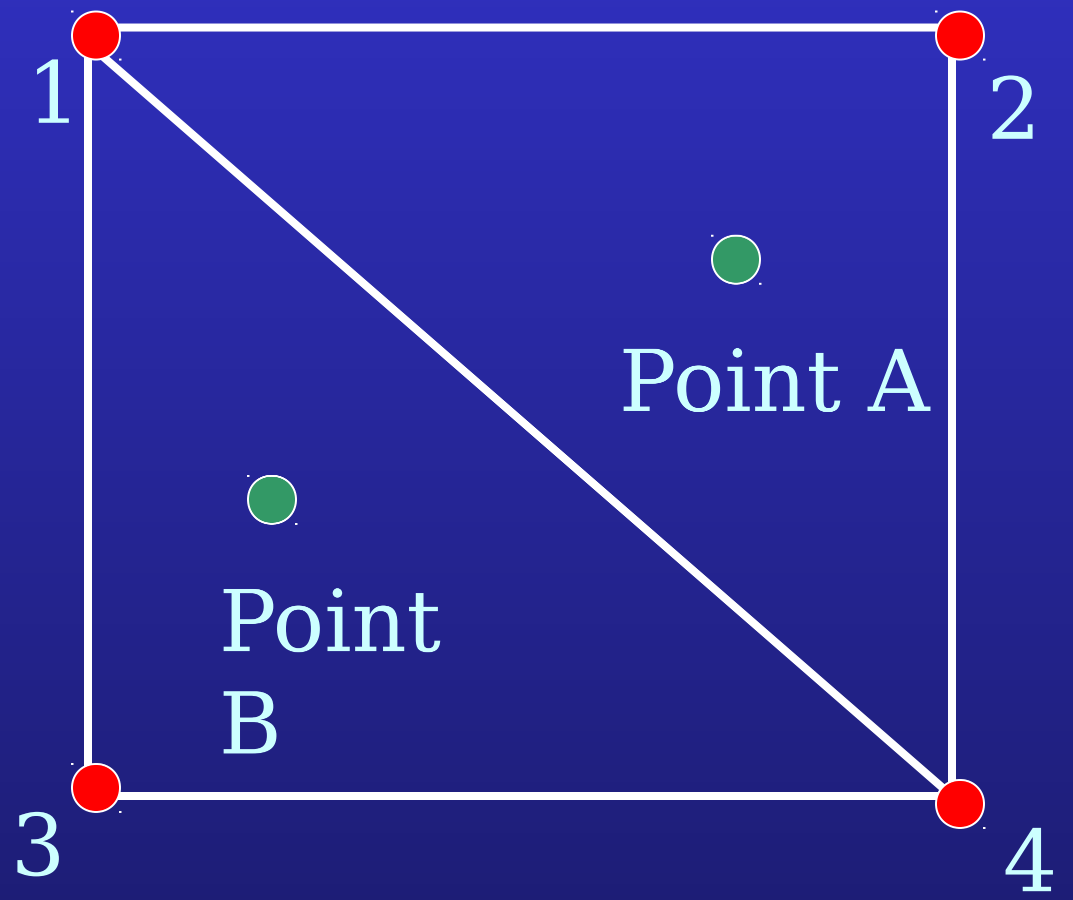



Circumcenter of the triangle

Pre-Sampling

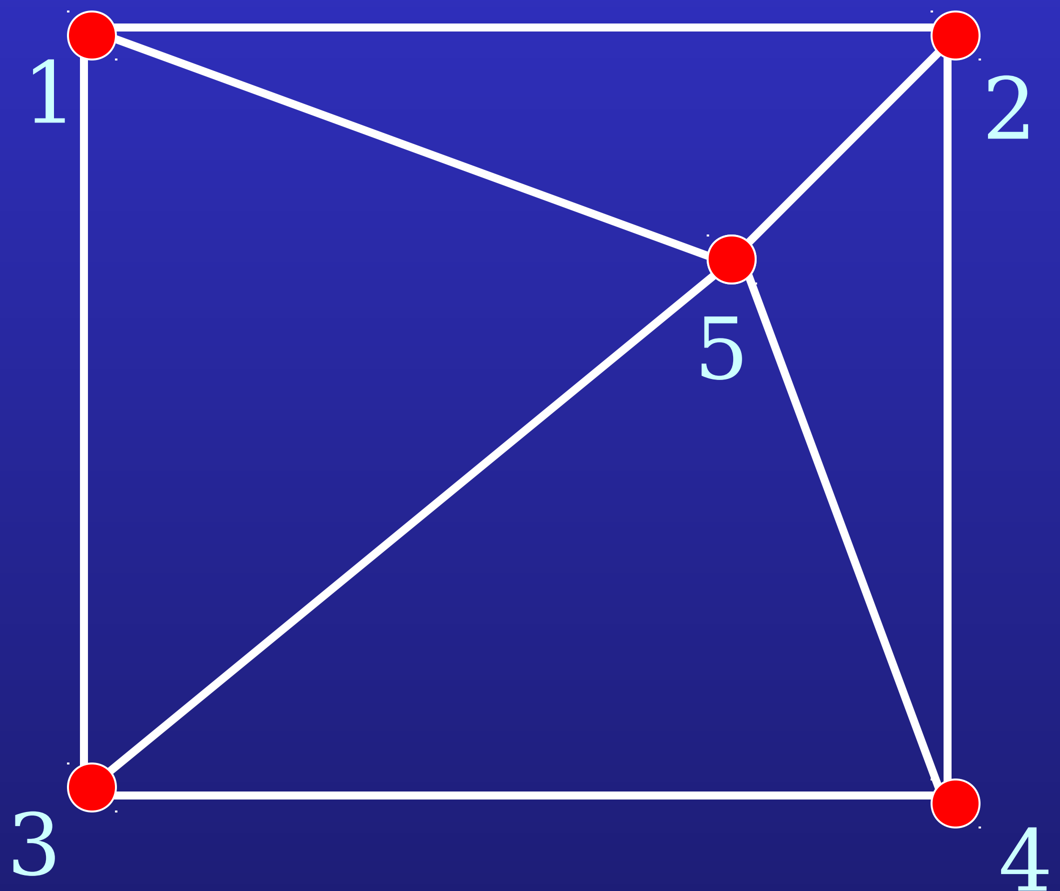
- Start with the minimal sample set in the domain.
- Generate the 2D Delaunay triangulation.
- Compute sphere parameters.
- While the sphere with ‘maximum radius’ has radius greater than a user specified parameter:
  - Append (*center*, radius) to the list of computed samples.
  - Update the delaunay triangulation by incrementally adding *center* and updating the center and radius of the affected triangles.

Pre-Sampling



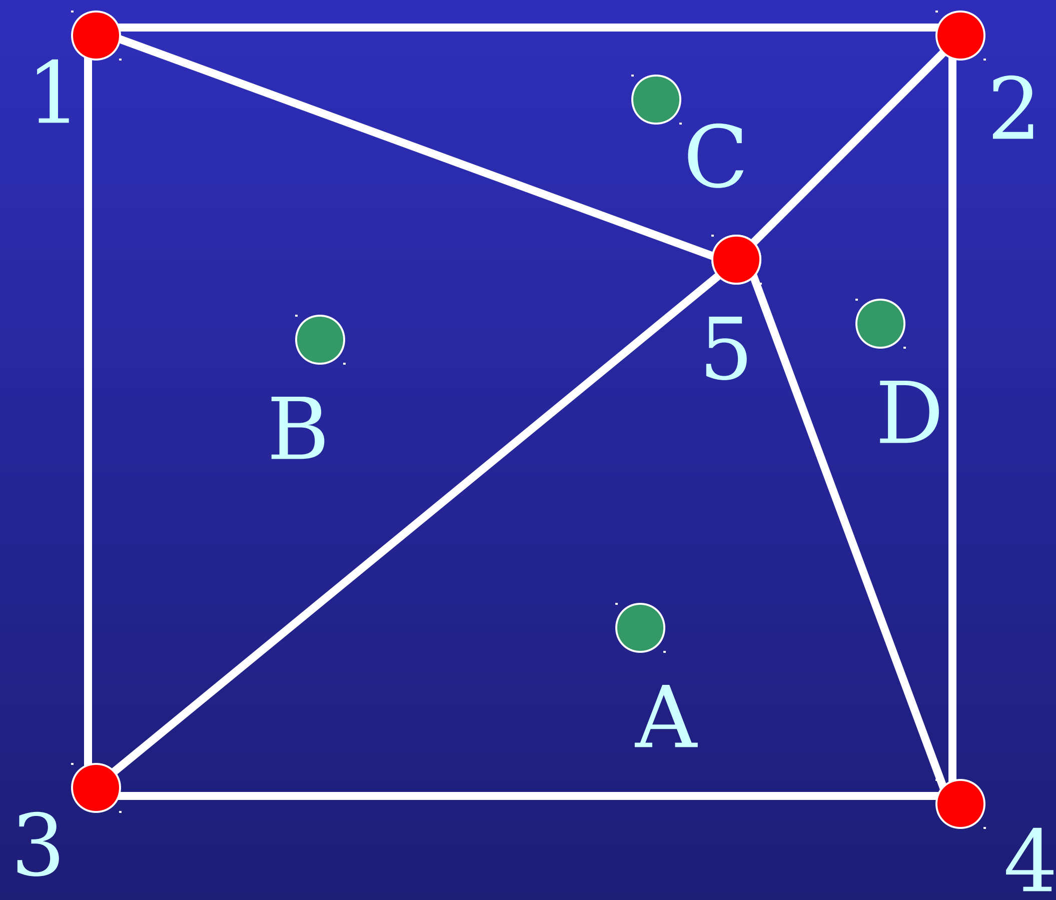
Circumcenter of the triangle

Pre-Sampling



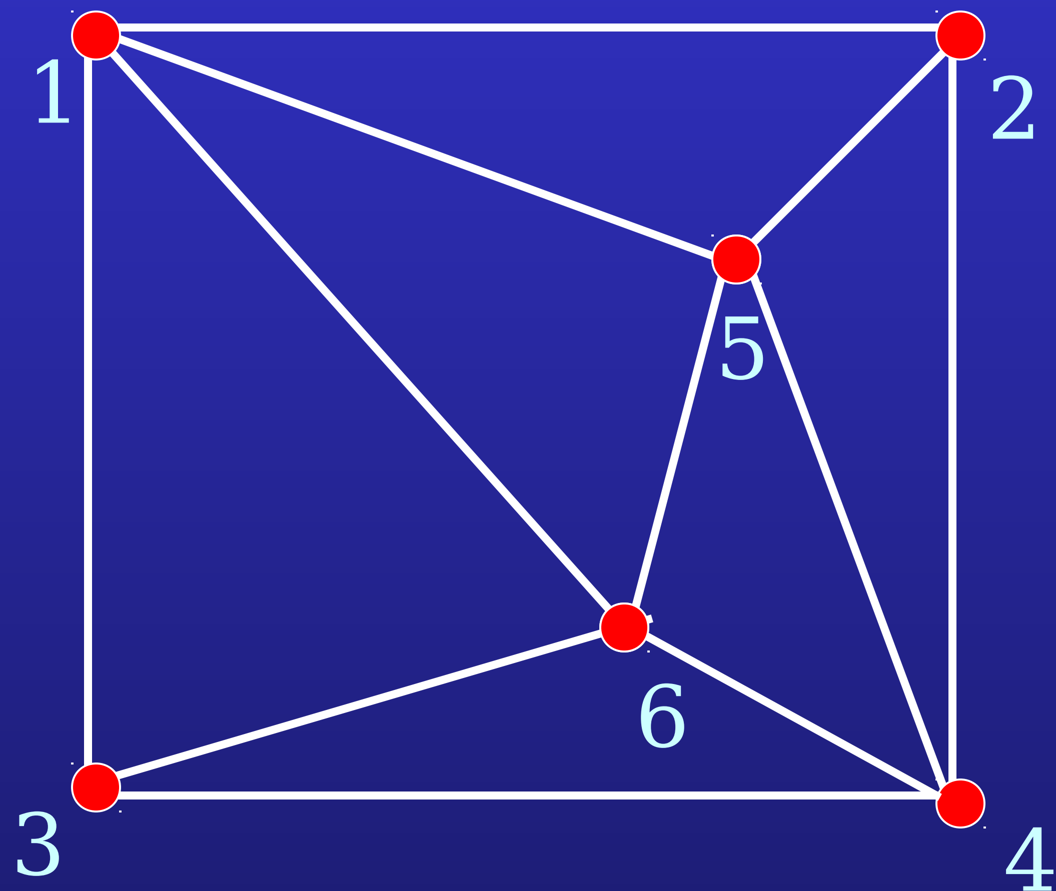
Domain Space

Pre-Sampling



Circumcenter of the triangle

Pre-Sampling



Domain Space

Pre-Sampling Properties

- Maximum deviation of a surface patch from the approximating spheres equals the radius of the sphere with the largest radius.
- Spheres drawn at the sampled points ensure a hole-free tiling of the surface patch.

What is stored ?

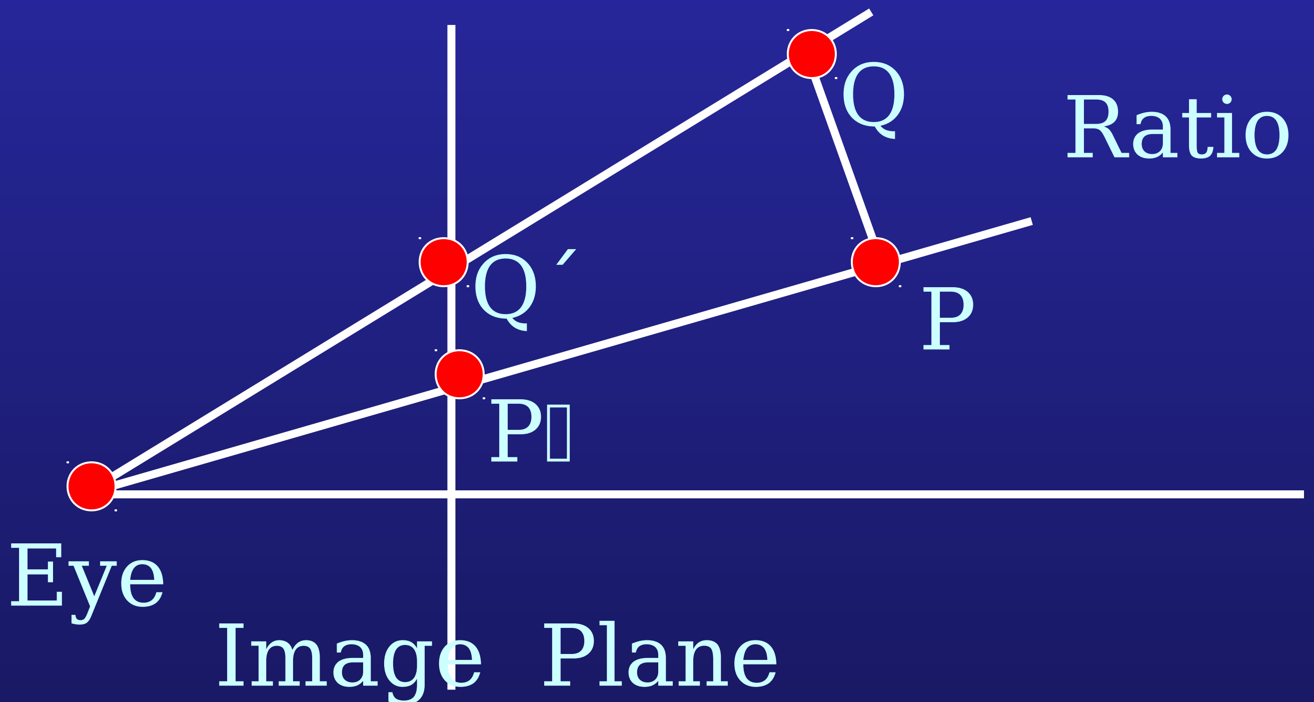
- Ordered set of (u,v) pairs
  - by decreasing deviation
- Deviation in **object space**
  - i.e., deviation after the sample is added
- 3-d Vertex
  - optional

Rendering Time Algorithm

1. Scaling Factor for a patch

Scaling Factor for a vector at point P is the

Minimum ratio of the length of the vector to its projected length on the image plane.



$$\text{Ratio} = \frac{|PQ|}{|P'Q'|}$$

1. Scaling Factor for a patch

- Pre-processing
  - Partition space
  - For each patch, use the partition containing it
  - If too many partitions for a patch, subdivide patch
- Run-time (for each frame)
  - Compute the scaling factor for each partition
  - Scaling factor a patch is that of its partition

2. Budget Allocation per patch

Question: Given a screen-space error (α), how to compute the number of points required for a given patch (F)?

Solution:

1. Compute the scaling-factor (γ).
2. Compute the object-space error = $\Delta=(\alpha * \gamma)$.
3. Find the index *j*, such that $\Delta F_{j-1} \geq \Delta > \Delta F_j$
4. Return (j).

Example

UV Values	P1	P2	P3	P4	P5	P6	P7	P8
Deviation	26	24	21	19	14	13	6	3

Example

UV Values	P1	P2	P3	P4	P5	P6	P7	P8
Deviation	26	24	21	19	14	13	6	3

Let $\Delta = 20$

Example

UV Values	P1	P2	P3	P4	P5	P6	P7	P8
Deviation	26	24	21	19	14	13	6	3

Let Δ = 20

Example

	P1	P2	P3	P4
UV Values				
Deviation	26	24	21	19

4 samples are chosen such that deviation is less than Δ (20)

2. Budget Allocation per patch

Assign a rendering size (*d*) of 1 initially for every point on each patch.

For every frame:

1. Compute the total points required ($C' = \Sigma C_i$).
2. If $C' < C$, then done.
3. Increment *d* by 1.
4. Go back to Step 1.

The above algorithm takes linear time to compute the right rendering size (and hence screen-space error).

2. Budget Allocation per patch (improved)

For every frame:

1. Assign the rendering size from the previous frame to every patch
2. Compute the total points required ($C' = \Sigma C_i$)
3. If $C' < C$, then for every patch:
   a. Decrease its rendering size by 1
   b. Recompute C'
   c. If $C' > C$ return
   d. Else go back to Step 3

2. Budget Allocation per patch (improved)

For every frame:

1. Assign the rendering size from the previous frame to every patch
2. Compute the total points required ($C' = \Sigma C_i$)
3. If $C' < C$, then for every patch:
   a. Decrease its rendering size by 1
   b. Recompute C'
   c. If $C' > C$ return
   d. Else go back to Step 3
4. If $C' > C$, then for every patch:
   a. Increment its rendering size by 1
   b. Recompute C'
   c. If $C' < C$ return
   d. Else go back to Step 4

2. Budget Allocation per patch (improved)

For every frame:

1. Assign the rendering size from the previous frame to every patch
2. Compute the total points required ($C' = \Sigma C_i$)
3. If $C' < C$, then for every patch:
   a. Decrease its rendering size by 1
   b. Recompute C'
   c. If $C' > C$ return
4. If $C' > C$, then for every patch:
   a. Increment its rendering size by 1
   b. Recompute C'
   c. If $C' < C$ return

The above is a 2n-time bounded algorithm exploiting the temporal coherence of the eye points.

3. Rendering Algorithm

For every patch:

1. Project the C_i on the screen using the computed rendering size (d).
2. In OpenGL:

glPointSize(d);

glColor3f(…);

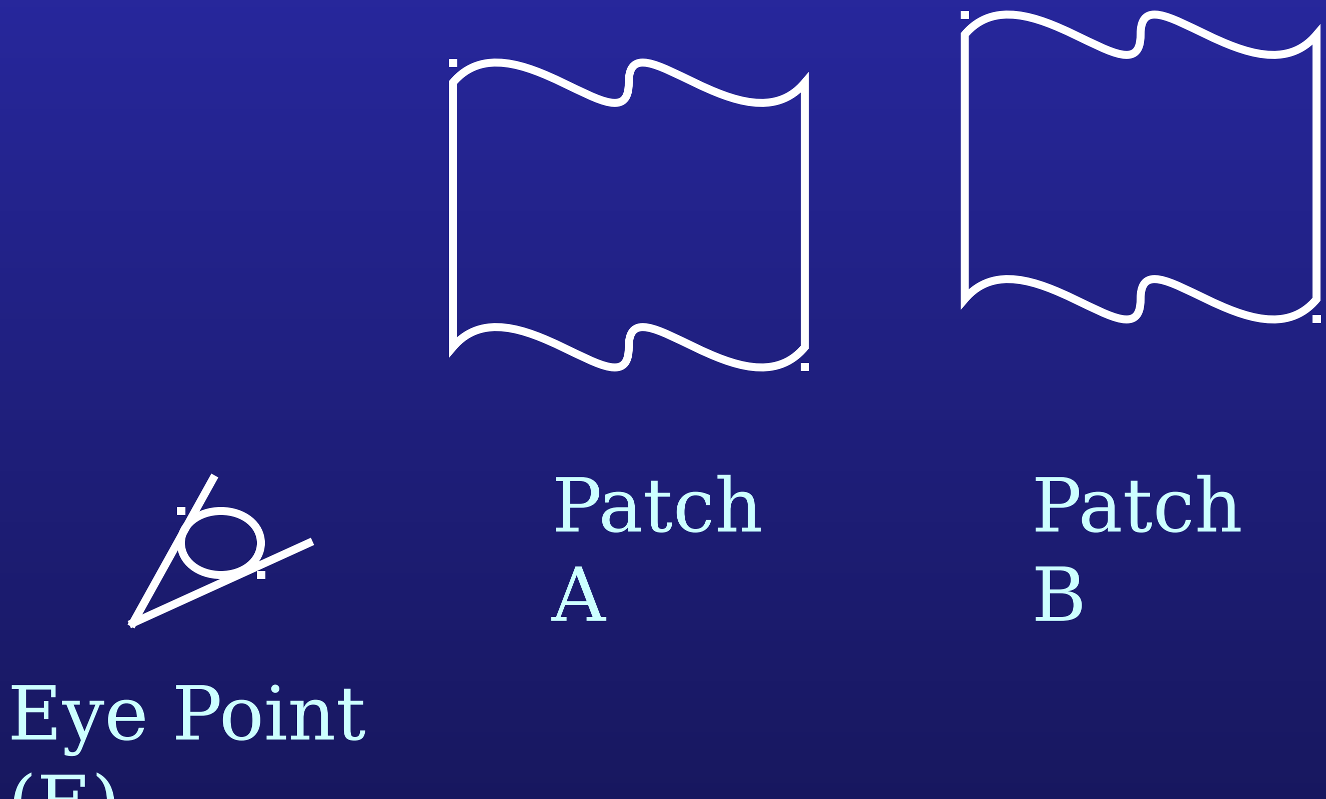
glNormalPointer(…);

glVertexPointer(…);

glDrawArrays(GL_POINTS, 0, C_i);

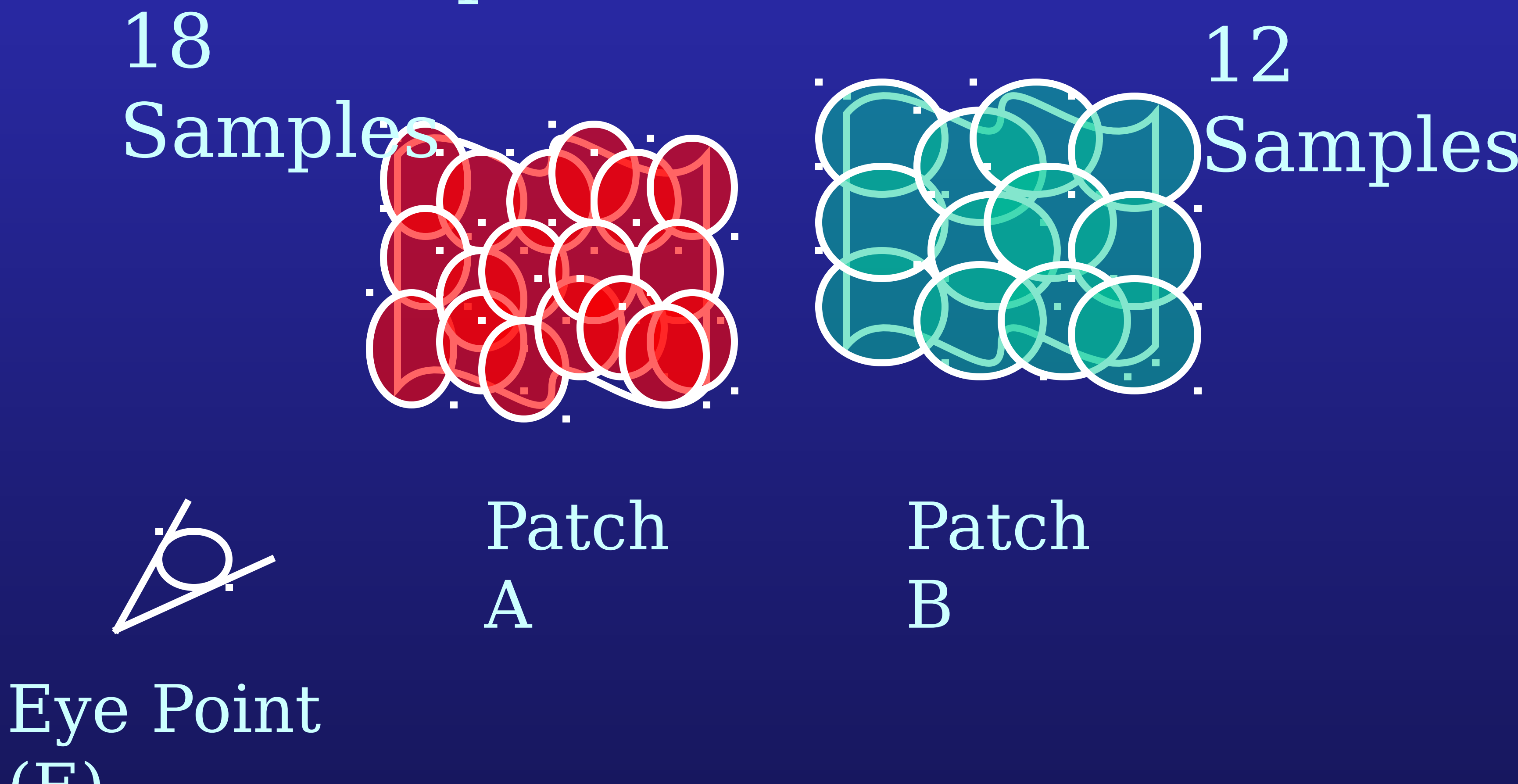
Example

Budget: 30 primitives



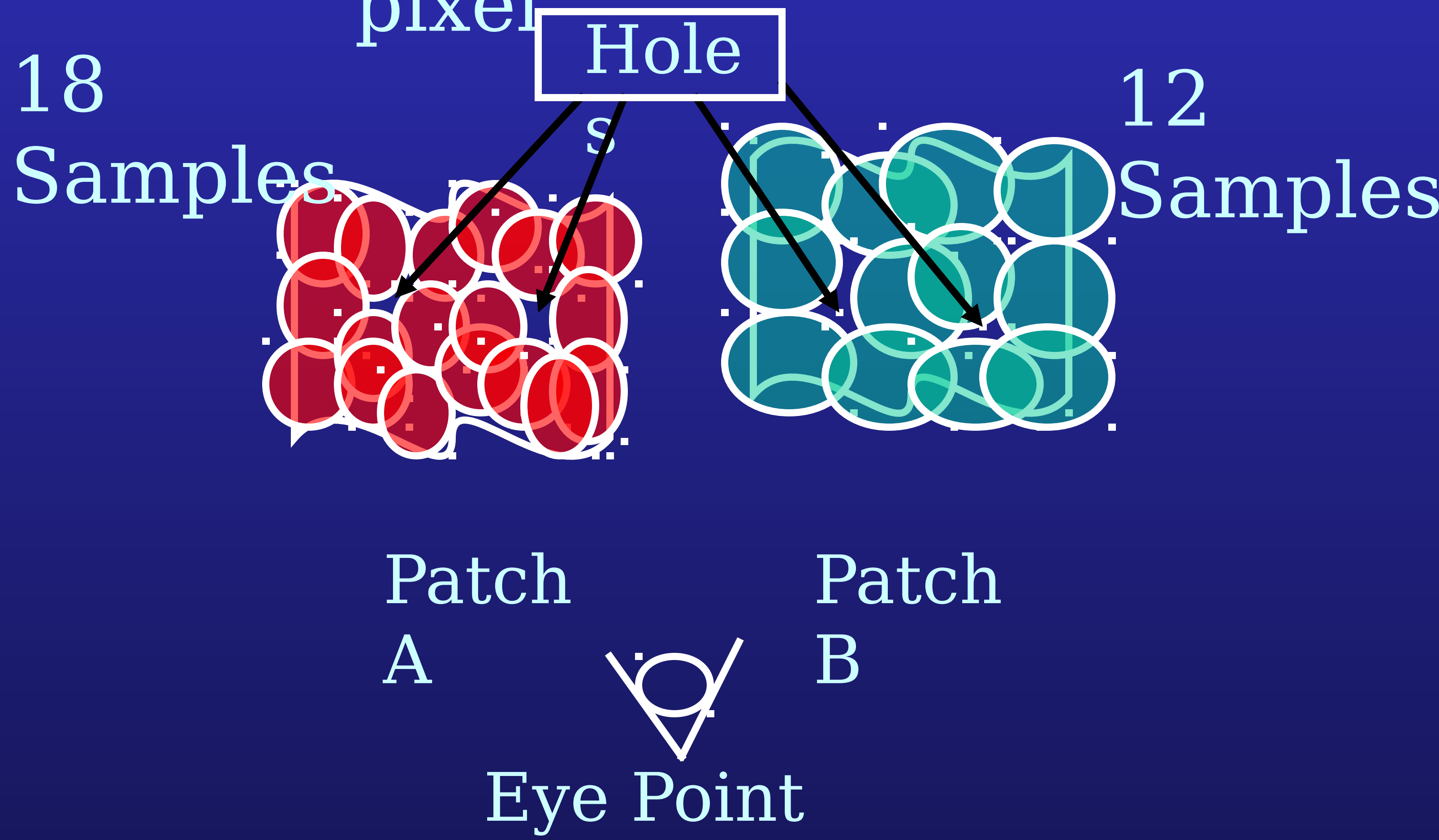
Example

Rendering Size: 1 pixel



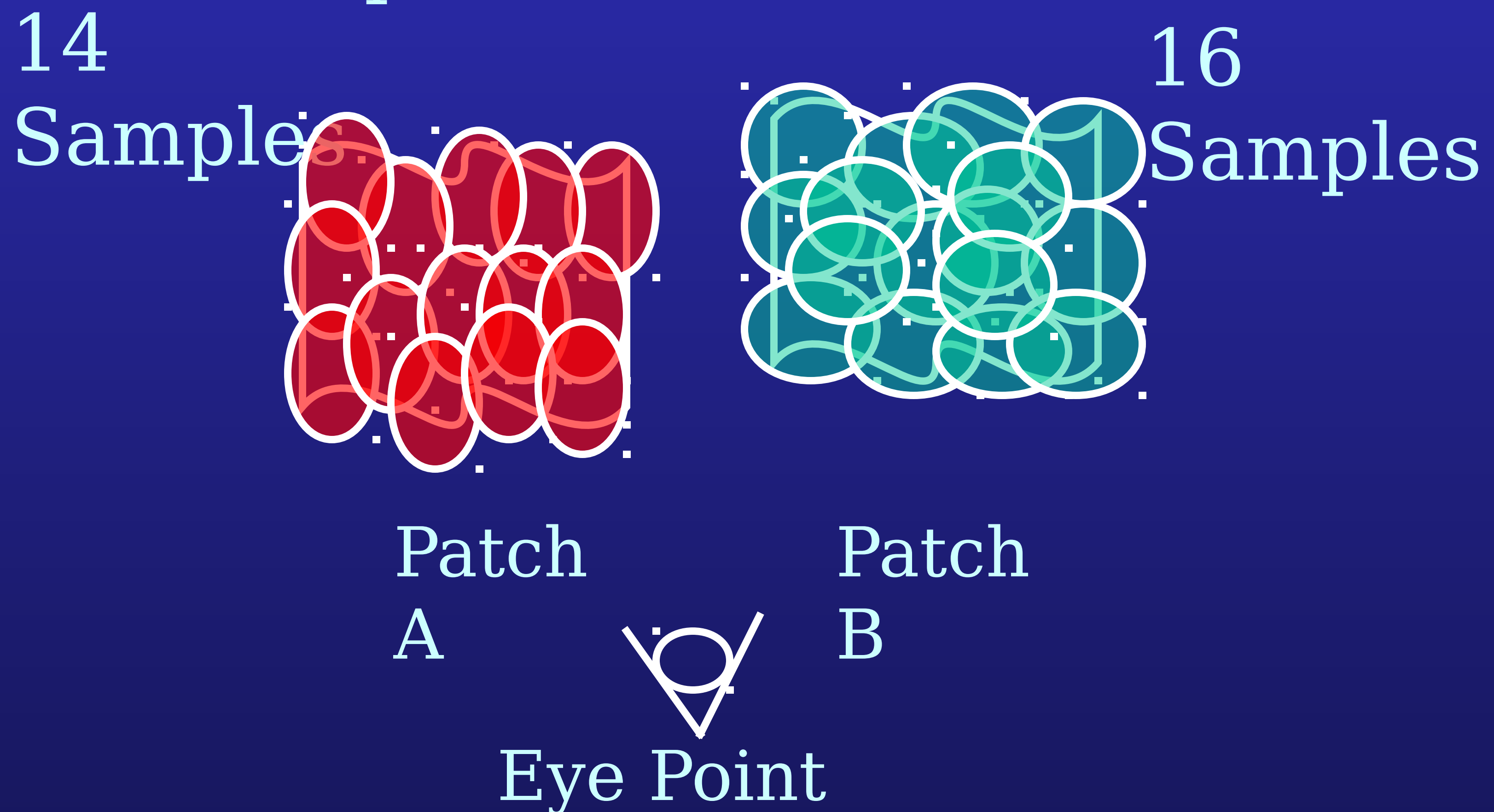
Example

Rendering Size: 1 pixel



Example

Rendering Size: 2 pixels



Results

Model	Patches	Pre-comp. Samples	Pre-proc. (mts)
Teapot	32	129,273	09
Goblet	72	123,396	15
Pencil	570	1,051,624	70
Dragon	5,354	1,473,961	96
Garden	38,646	1,231,200	82

Pre-Sampling Performance

Results

Model	Patches	Points per Frame	Time in Software	Frame Rate
Teapot	32	90,000	0.3%	31
Goblet	72	100,000	0.5%	34
Pencil	570	70,000	3.4%	23
Dragon	5,354	50,000	11.9%	20
Garden	38,646	50,000	19.1%	7

Run-time Performance

Budget Sampling of Parametric Surface Patches

Jatin Chhugani and Subodh Kumar
Johns Hopkins University

Conclusions

- View-dependent algorithm for distributing points across patches
- Provides guaranteed primitive budget
- Applicable to class of parametric surfaces
- Towards real-time spline surface rendering

Acknowledgements

- Shankar Krishnan
- Jonathan Cohen
- Budirijanto Purnomo
- Lifeng Wang
- UBC Modeling group
- Alpha 1 Modeling system
- National Science Foundation
- Link Foundation

The End.

Splines

- Non-Uniform Rational B-Spline (NURBS)
- Bezier patch (rational)

  Degree m x n

  Domain space (u, v) ε [0,1] x [0,1]

  For $0 \le i \le m, 0 \le j \le n,$

  Control points : p_{ij}

  Weights: w_{ij}

$$F(u,v) = \frac{\sum_{i=0}^{m} \sum_{j=0}^{n} w_{ij} \mathbf{p}_{ij} B^m_i(u)\, B^n_j(v)}{\sum_{i=0}^{m} \sum_{j=0}^{n} w_{ij} B^m_i(u)\, B^n_j(v)}$$

where

Bernstein function $B^n_i(t) = \binom{n}{i} t^i (1-t)^{n-i}$

Knapsack Formulation

- Cost/Benefit Formulation (Funkhouser [`93])
  - Maximize Σ (*Benefit*)

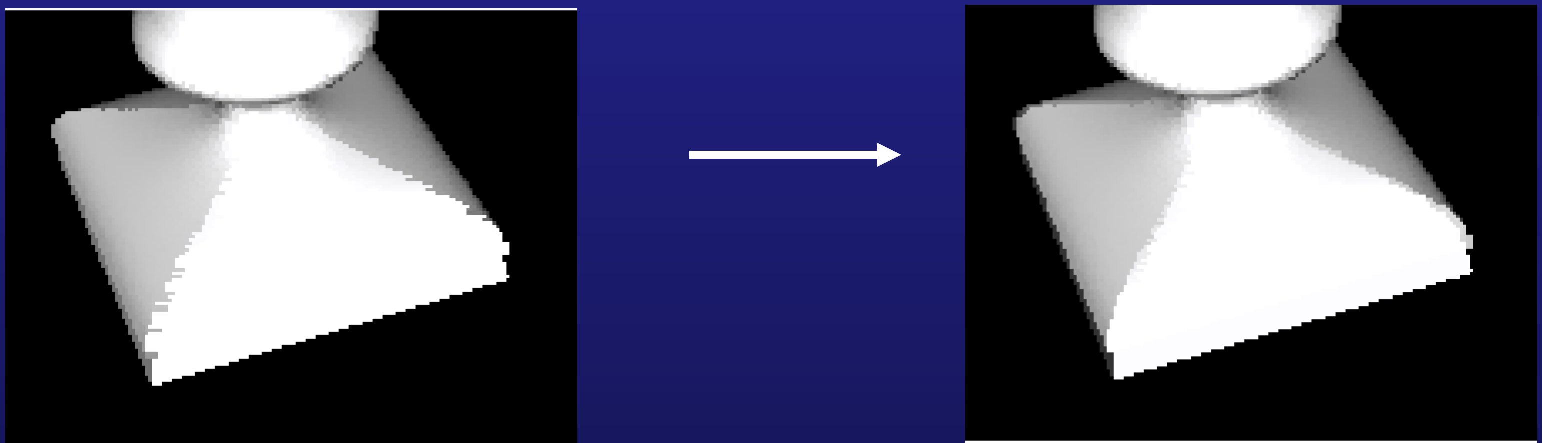
    s.t. Σ (*Cost*) < *Frame RenderingTime*

- Granulartiy of our problem is much finer.
  - Knapsack too slow

Visual Artifacts

Aliasing effects across the boundary of a patch

Reduction in artifacts by averaging normals across patch boundaries



Results
[10,000 points]

Results
[20,000 points]

Results
[30,000 points]

Results
[40,000 points]

Results
[50,000 points]

Results
[60,000 points]

Results
[70,000 points]

Results
[80,000 points]

Results
[90,000 points]

Results
[100,000 points]